

Phenomenological High Precision Neutron-Proton Delta-Shell Potential

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We provide a successful fit for proton-neutron scattering below pion production threshold up to LAB energies of 350MeV. We use seven high-quality fits based on potentials with different forms as a measure of the systematic uncertainty. We represent the interaction as a sum of delta-shells in configuration space below the 3fm and a charge dependent one pion exchange potential above 3fm together with magnetic and vacuum polarization effects. Special attention is paid to estimate the errors of the phenomenological interaction.

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The study of the NN interaction has been a central and recurrent topic in Nuclear Physics for many years (see e.g. [1, 2] and references therein). The standard approach to constrain the interaction is to analyze NN scattering data below pion production threshold and to undertake a partial wave analysis (PWA), the quality of the fit being given by the χ^2/dof value. Only by the mid 90's was it possible to fit about 4000 selected NN scattering data after discarding about further 1000 of 3σ inconsistent data with a $\chi^2/\text{dof} \lesssim 1$ and incorporating charge dependence (CD) for the One Pion Exchange (OPE) potential as well as magnetic and vacuum polarization effects [3]. This benchmark partial wave analysis (PWA) was carried out using an energy dependent potential for the short range part for which nuclear structure calculations become hard to formulate. Thus, energy independent *high quality* potentials were subsequently produced with almost identical $\chi^2/\text{dof} \sim 1$ for a gradually increasing database [4–7]. While any of these potentials provides individually satisfactory fits to the available data it is striking that there are no published error estimates of the potential parameters. Moreover it should also be noticed that the existing high-quality potentials are different in their specific form; they range from local to non-local in different versions of nonlocality. Thus, scattering phase-shifts and observable amplitudes are not identical and in fact the existing set of high quality potentials as a whole provides a distribution of scattering observables accounting for systematic uncertainties in addition to the statistical uncertainties obtained from the fitted data for each individual potential. Given the fact that these interactions are just constrained to the elastic scattering data (and eventually to the deuteron) which go up to the pion production threshold, one is physically probing the interaction with a resolution not finer than the shortest de Broglie wavelength $\Delta\lambda = \hbar/\sqrt{M_N m_\pi} \sim 0.5\text{fm}$. Thus, for practical purposes it may be advantageous to consider *coarse grained* interactions. This is actually the physics underlying the so-called V_{lowk} approach [8] in which an effective interaction in a restricted model space is built. By starting from *different* high-quality potentials with a common charge dependent OPE interaction the CM-momenta above $\Lambda \sim \sqrt{M_N m_\pi}$ are eliminated by a suitable transformation and a remarkable

universal interaction is obtained for $p \leq \Lambda$. Many of the applications of such an appealing interaction have recently been reviewed [9].

On the other hand, when switching from the NN problem to the many body nuclear problem the features and the form of the interaction are relevant in terms of computational cost and feasibility. The lack of knowledge of a precise potential form with finer resolution than $\Delta r \sim 0.5\text{fm}$ suggests to search for a description of scattering data directly in terms of a coarse grained potential sampled at some sensible “thick points”. Any sampling procedure necessarily redistributes the interaction strength and smoothes the potential as compared to the zero resolution limit $\Delta r \rightarrow 0$ implicit in most potential approaches and generating the troublesome short distance cores. Of course the optimal way to sample the interaction is to provide an acceptable χ^2 -fit with the minimal number of sampling points; by implementing this minimal sampling we just prevent statistical correlations between the strengths of the potential at the sampling points.

In the present work we provide another high-quality potential accommodating these desirable features and (unlike the previous approaches) we undertake an analysis of the systematic errors. We do this by assuming that the systematic error inherent to any specific choice of the potential form corresponds to individual uncorrelated measurements. Hence we may invoke the central limit theorem to undertake the traditional statistical treatment to the mean average and the corresponding standard deviation of Refs. [3–7] without any further ado. We will use this compilation as our database.

A convenient representation to sample the the short distance component of the NN interaction was already suggested by Aviles [10] almost 40 years ago in terms of delta-shells which for any partial wave $^{2S+1}(L, l')_J$ we take as

$$V_{l,l'}^{JS}(r) = \frac{1}{2\mu_{np}} \sum_{n=1}^N (\lambda_n)_{l,l'}^{JS} \delta(r - r_n) \quad r \leq r_c, \quad (1)$$

with μ_{np} the reduced pn-mass and r_c to be specified below. In the spirit of Refs. [3–7], for $r \geq r_c$ we use the well known long-distance tail of the NN potential

$$V(\vec{r}) = V_{CD}(\vec{r}) + V_{OPE}(\vec{r}), \quad r > r_c, \quad (2)$$

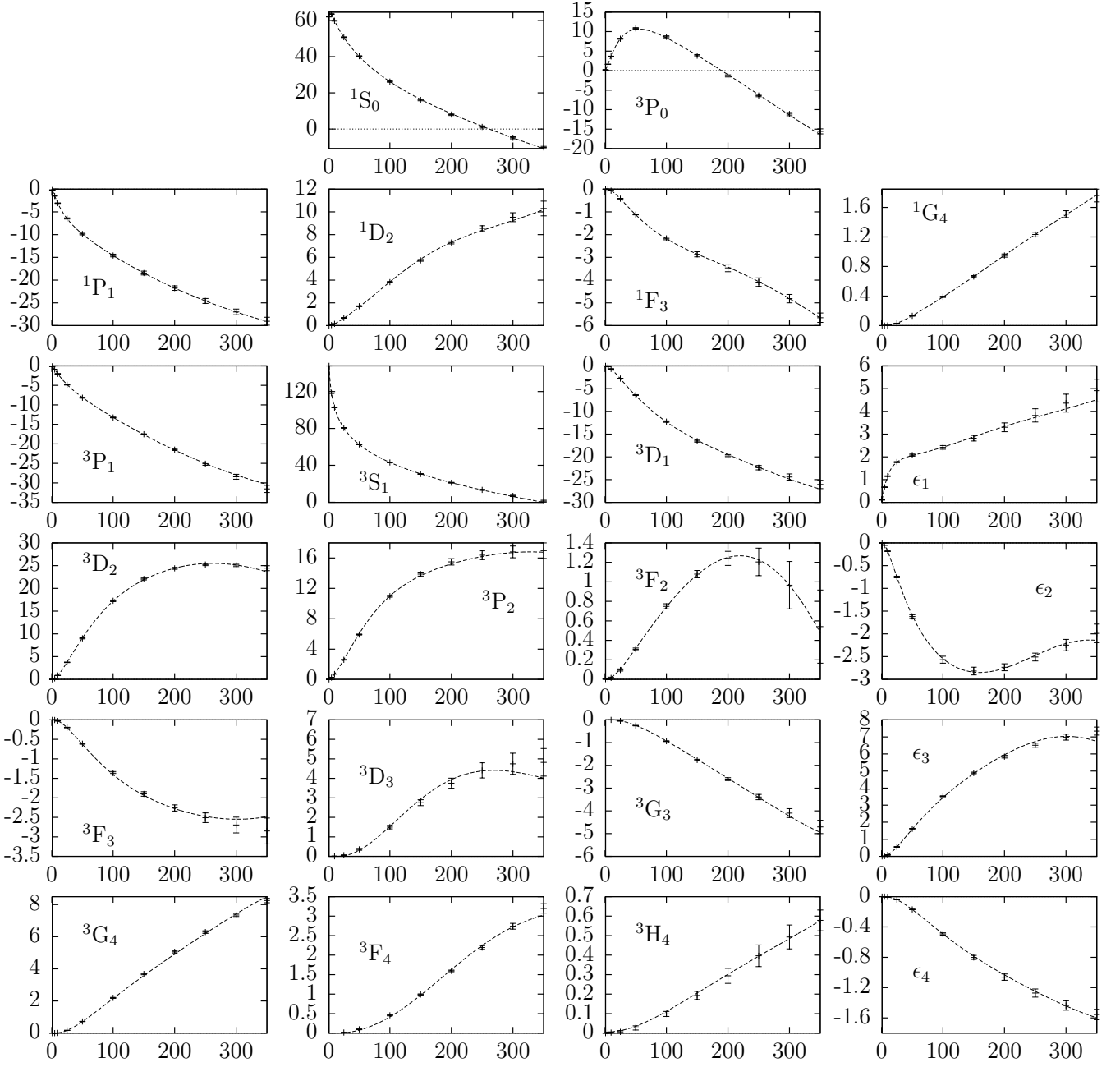


FIG. 1. np phase shifts in degrees with $J \leq 4$ as a function of the LAB energy in MeV. The dashed line is calculated with the fitted potential and the points with error bars represent the mean value and standard deviation of the compilation of the PWA [3] and the six high quality potentials [4–7]. We fit the energies $E_{\text{LAB}} = 1, 5, 10, 25, 50, 100, 150, 200, 250, 300, 350$ MeV.

where V_{CD} is the charge-dependent (CD), magnetic and vacuum polarization potential of Ref. [5], and V_{OPE} is the one-pion-exchange potential.

The solution of the corresponding Schrödinger equation in coupled channels is straightforward; for any $r_n < r < r_{n+1}$ with $r_N < r_c$ we have free solutions and log-derivatives are discontinuous at the r_n -points so that one generates an accumulated S-matrix at any sampling point providing a discrete

version of Calogero's variable phase equation [11]. Although this potential is formally local, the fact that we are coarse graining the interaction enables to encode efficiently nonlocalities operating below the finest resolution Δr . Of course, once we admit that the interaction below r_c is unknown there is no advantage in prolonging the well-known charge-dependent OPE tail and other electromagnetic effects for $r < r_c$. The low energy expansion of the discrete variable phase equations

TABLE I. Fitting delta-shell parameters $(\lambda_n)_{l,l'}^{JS}$ (in fm^{-1}) with their errors for all states in the JS channel and the corresponding χ^2 -value for $J \leq 4$ in np scattering. We take $N = 5$ equidistant points with $\Delta r = 0.6\text{fm}$. — indicates that the corresponding $(\lambda_n)_{l,l'}^{JS} = 0$.

Wave	λ_1	λ_2	λ_3	λ_4	λ_5	χ^2/dof
1S_0	2.12(7)	-0.987(9)	—	-0.087(2)	—	0.3476
3P_0	—	1.27(4)	-0.43(1)	—	-0.035(2)	1.1803
1P_1	—	1.23(2)	—	0.079(4)	—	0.0088
3P_1	—	1.33(2)	—	0.053(2)	—	0.4371
1D_2	—	-0.64(2)	—	-0.082(2)	—	1.5671
3D_2	—	—	-0.597(8)	-0.08(1)	-0.051(4)	0.6431
1F_3	—	2.6(6)	—	0.068(7)	—	0.1603
3F_3	—	—	—	0.060(2)	—	0.7759
1G_4	—	—	-0.22(2)	—	-0.0137(9)	0.8090
3G_4	—	—	—	-0.267(3)	—	1.6647
3S_1	1.44(3)	-0.40(1)	—	-0.064(3)	—	0.4750
ϵ_1	—	-1.67(1)	-0.412(8)	-0.200(6)	-0.031(3)	
3D_1	—	—	0.52(2)	—	0.041(3)	
3P_2	—	-0.413(6)	—	-0.0398(9)	—	
ϵ_2	—	0.65(1)	—	0.105(2)	—	0.7125
3F_2	—	—	0.11(3)	-0.069(6)	—	
3D_3	—	-0.13(4)	—	0.010(3)	—	
ϵ_3	—	—	-0.31(3)	-0.323(7)	—	
3G_3	—	—	—	0.115(6)	—	1.7552
3F_4	—	-0.47(6)	—	-0.051(1)	—	1.4917
ϵ_4	—	—	—	0.107(3)	—	
3H_4	—	—	—	-0.039(6)	—	

was used in Ref. [12] to determine threshold parameters in all partial waves. The relation to the well-known Nyquist theorem of sampling a signal with a given bandwidth has been discussed in Ref. [13]. Some of the advantages of using this simple potential for Nuclear Structure calculations as well as the connection to the V_{lowk} approach have been spelled out already [14].

We use the standard LAB energy values usually listed in the high-quality potentials, namely $E_{\text{LAB}} = 1, 5, 10, 25, 50, 100, 150, 200, 250, 300, 350\text{MeV}$ and fit to the mean phase-shift values at those energies with an error equal to the standard deviation. In this way we account for the systematic errors due to the different representations of the potentials [3–7]. We find that they are generally larger than those quoted by the original PWA where only statistical uncertainties were explicitly discussed for a *fixed* potential form [3]. With these data sets and the given energies we undertake a phase-shift fit and determine errors using the standard covariance matrix.

As expected, we need at most $N = 5$ sampling points which for simplicity are taken to be equidistant with $\Delta r = 0.6\text{fm}$ between the origin and $r_c = 3\text{fm}$. This is the minimal number which provides an acceptable fit to the data compiled from Refs. [3–7]. Our results for the pn phase-shifts for all par-

tial waves with total angular momentum $J \leq 4$ are depicted in Fig. 1. The fitting parameters $(\lambda_n)_{l,l'}^{JS}$ entering the delta-shell potentials, Eq. (1), are listed in Table I with their deduced uncertainties. Of course, a definitive assessment on systematic errors would require testing *all possible* potential forms. Thus, the errors will generally be larger than those estimated here. We find that correlations among the different $(\lambda_n)_{l,l'}^{JS}$ values within a given partial wave channel are unimportant, and hence these parameters are essentially independent from each other. This is a direct consequence of our strategy to minimize the number of sampling points. We find that introducing more points or equivalently reducing Δr generates unnecessary correlations. Also, lowering the value of r_c below 3fm , requires overlapping the short-distance potential, Eq. (1), with the CD-OPE plus em corrections.

We determine the deuteron properties by solving the bound state problem in the $^3S_1 - ^3D_1$ channel using the corresponding parameters listed in Table I. The predictions are presented in table II where our quoted errors are obtained from propagating Table I. The comparison with experimental values or high quality potentials where the binding energies are used as an input is satisfactory. This is partly due to the fact that theoretical errors are about 10%. Of course, one may improve on this by using the deuteron binding energy as an input as in

TABLE II. Deuteron static properties compared with empirical values and high-quality potentials calculations

	Delta Shell	Empirical[15–20]	Nijm I [4]	Nijm II [4]	Reid93 [4]	AV18 [5]	CD-Bonn [6]
$E_d(\text{MeV})$	2.2(2)	2.224575(9)	Input	Input	Input	Input	Input
η	0.025(2)	0.0256(5)	0.02534	0.02521	0.02514	0.0250	0.0256
$A_S(\text{fm}^{1/2})$	0.88(3)	0.8781(44)	0.8841	0.8845	0.8853	0.8850	0.8846
$r_m(\text{fm})$	1.97(8)	1.953(3)	1.9666	1.9675	1.9686	1.967	1.966
$Q_D(\text{fm}^2)$	0.272(9)	0.2859(3)	0.2719	0.2707	0.2703	0.270	0.270
P_D	5.7(2)	5.67(4)	5.664	5.635	5.699	5.76	4.85
$\langle r^{-1} \rangle(\text{fm}^{-1})$	0.45(1)			0.4502	0.4515		

Refs. [3–7].

Fitting to phase-shifts to some accuracy needs not provide angle dependent scattering amplitudes to the same accuracy. This is often the case when the form of the potential is kept fixed, so that the channel by channel fit is usually taken as a first step which is afterwards refined by a full fledged analysis of differential or scattering observables and polarization data [3–7]. To check this issue we proceed as follows. The complete on-shell np scattering amplitude contains five independent complex quantities, which we choose for definiteness as the Wolfenstein parameters and denote generically as a 10-dimensional array (a_1, \dots, a_{10}) , which could be determined directly from experiment as suggested recently [21]. We follow an alternative procedure and construct, out of the high-quality analyses, the corresponding mean value of the Wolfenstein parameters, $\bar{a}_i(E_{\text{LAB}}, \theta)$, with their corresponding standard deviations, $\Delta a_i(E_{\text{LAB}}, \theta)$, for any given LAB energy and scattering angle θ as

$$\chi^2(E_{\text{LAB}}, \theta) = \sum_{i=1}^{10} \left[\frac{\bar{a}_i(E_{\text{LAB}}, \theta) - a_i(E_{\text{LAB}}, \theta)}{\Delta a_i(E_{\text{LAB}}, \theta)} \right]^2 \quad (3)$$

The total χ^2 value is obtained as an average over the chosen reference energies $E_{\text{LAB}} = 1, 5, 10, 25, 50, 100, 150, 200, 250, 300, 350 \text{ MeV}$ and a dense 1° -equidistant sampling of θ -values. The result is

$$\chi^2/\text{dof} = 0.78$$

which is equivalent to carry a complete χ^2 -fit to the mean average scattering amplitude. Again, this is a virtue of the minimal coarse graining of the interactions which adds just the needed number of *independent* parameters.

To summarize, we have determined a high-quality neutron-proton interaction which is based on a few delta-shells for the lowest partial waves in addition to charge-dependent electromagnetic interactions and one pion exchange and provides a good starting point for Nuclear Physics applications. We provide error estimates on our fitting parameters accounting both for systematic and statistical uncertainties of present day high-quality analyses of neutron-proton scattering data. Deuteron properties are compatible with experimental or recommended values. Our method allows to coarse grain long-range Coulomb interactions in a rather natural way and hence to discuss proton-proton scattering data.

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